



1. Let  $S$  be the set of all integers of the form  $P^2 - 1$  where  $P$  is a prime number greater than 5. Let  $N$  be the largest integer that divides every member of  $S$ . Find, with proof, the value of  $N$ .
2. In triangle  $ABC$ ,  $(\cos B)(\cos C) = \cos A$ . Find, with proof, the numerical value of  $(\tan B)(\tan C)$ .
3. Suppose that  $n + 1$  boys are lined up shoulder-to-shoulder from left to right in a straight line. Prove that it is always possible to select  $n + 1$  boys to take one step forward so that, going from left to right, their heights are either consistently increasing or consistently decreasing.
4. The lengths of the sides of triangle  $ABC$  are in the ratio of 4:5:6. The bisector of the largest angle of the triangle is drawn. Prove that one of the two triangles formed also has sides whose lengths are in the ratio of 4:5:6.
5. All the factors of the polynomial  $P(x) = x^3 + ax^2 + bx + b$  are linear with integer coefficients, and neither  $a$  nor  $b$  are zero or one. Find all possible pairs  $(a, b)$ , and prove that you have found them all.

3 3 ± 3 6LQFH 3 LV D SULPH DQG 3 ! 3 LV RG  
DQG 3 PXVW EH FRQVHFXWLYH HYHQ QXPEHUV 7K  
PXOWLSOH RI3 ZKLV KYRWDQOH E\ 3\$DQB 3VLQFHUH ±  
WKUHH FRQVHFXWLYH LQWHJHUV RQH RI WKHP PXVW  
HLWKHU ± PXVW EH D PXOWLSOHGLYLVL7KHU HEIRUH  
,I 33 ,I 3 3 6LQFH LV WKH JFG RI DQ  
N=24 ODUJHVW

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7KHUHIRUH FRV% FRV& VLQ% VLQ& VR WKDW

$$\frac{VLQ}{FRV} \quad WDI$$

\$VVXPH LW LV LPSRVVLEOH WR ILQG Q ER\ V LQ DVF  
:H ZLOO VKRZ WKDW LW LV WKHQ SRVVLEOH WR ILQG Q  
6WDUWLQJ ZLWK WKH ILUVW ER\ ZH IRUP D 'FOXE' LQ V  
ER\ WDOOHU WKDQ KLP WKH ILUVW ER\ WDOOHU WKDQ  
WKDQ Q ER\ V LQ WKLV FOXE 1H[W ZH FUHDWH D VHFRQ  
WKH ILUVW FOXE DQG FKRRVLQJ WKH ILUVW ER\ ZKR LV  
ILUVW ER\ LQ WKH VHFRQG FOXE DQG FRQLQXLQJ LQ W  
ER\ V LQ WKH VHFRQG FOXE 1RWH WKDW HDFK ER\ LQ V  
ER\ LQ WKH ILUVW FOXE ZKR LV WDOOHU WKDQ KLP RW  
1RZ IURP DPRQJ WKH ER\ ZKR DUH QRW LQ HLWKHU R  
FOXE VWDUWLQJ ZLWK WKH ILUVW ER\ QRW LQ WKH ILU  
WKDQ Q ER\ V LQ WKH WKLV FOXE DQG HDFK RI WKHVH  
ZKR LV WDOOHU WKHQ KLP :H FRQLQXH LQ WKLV ZD\  
PRUH WKDQ Q PHPEHUV DQG HDFK PHPEHU RI HDFK FO  
FOXE

7KHUH DUH DWPRUW LQ WKHVH Q FOXEV VR QRZ OHW X  
DQ\ RI WKHVH FOXEV +H PXVW IROORZ D WDOOHU ER\  
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/HW WKH VLGHV RI WULDQJOH \$%& KDYH OHQJWKV  
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LV WKH ODUJHVW DQJOH% L'QVZUWKHF DQJOH ELVHFWRU  
\$'DW SRLQW ' 6LQFH WKH OHQJWKV RI FRUUVSRQGLC  
VLPLODUDWHLSDUJSDUJWLRQDO ZH QHHG RQO\ SURYH  
WKH WWRKOHU DQJOHV IRUP\$%& LV VLPLODU WR  
6LQFH ERWK WULDQJOHV \$%& DQG \$' % DUH DFXWH ZH  
WKDWWWKLH DQJOH ZRDUH VLPLODU

0HWKRG

8VLQJ WKH DQJOH-ELVHFWRU ZKLFK RUHP

- 6LQFH WKH UDWLR RI \$% WR \$' LV

$\frac{\$ \%}{\$ '}$   $\frac{\$ \&}{\$ \%}$  7KX\$' % DQ\$%& KDYH WZR SDLUV

RI SURSRUWLRQDO VLGHV DQG VKDUH WKH LQFOXGH  
DQJOH \$ 7KHUHIRUH WKH WZR WULDQJOHV DUH VLP  
SURY% QJOVR KDV VLGBUHZKRVH OHQJWKV  
LQ WKH UDWLR RI

0HWKRG

6LQFH DQ\$%& VKDUH DQJOH \$ ZH QHHG RQO\ ILQG  
RQH DGGLWLRQDO SDLU RI FRQJUXHQW DQJOHV \$Q  
FDQQRW EH FRQJUXHQW WR DQJOH \$%& , LV LV F  
WR DQJOH & WKHQ WULDQJOH %' & ZRXOG KDYH WR  
7QLRQD KDSSHQ LI WKH PHDVXUH RI DQJOH \$%&  
ZLHWWKDW RI DQJOH & \$

8VLQJ WKH /D\$%& & RVLQH V RQ

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7KXV WULDQJOH \$%& LV VLPLD\$% DVV WU KIDDQV DGH\$' %Z K  
OHQDULQ WKH UDWLR RI

