Kennesaw

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Q DG GL WL RQ WR VF RU LQ JZ KW WK HQ D UH RV SX RQ LV HQ V LE\
F RQ VL GH UD WL RQ WR ZH JD QF EH JV LY HS QO LF LW \ RU LJ QD O

Calculators are NOT permi

7 K H HJ XD Wa xR Q U H SU H V H Q W V D S D U D E a R Z ÌÒ R U D O O U

L f x $\frac{x}{x}$ DQ G f_n Lx L f f_n x

nI R U « &R P S X W fH Z L W K S U R R I

&R Q VL GH U WK H WK U H H DQ G P E H Z K H U H D S R VL WL Y H L

D) L Q G Z L W K S U R R I R I D Z K L F K O X H Y W K U H H Q X P E H U V D

E 3U R Y H WK DW WK H U H DU H I Q I Z K L W K H Q \R P D H Q R I Y W K H W
LV D SU L P H Q X P E H U

WK H F R U U H F W U R R P N H \

,Q WK H I L J X U H $ % & ' L V D V T X DU H 0 L V WK H P L G S R I
DQ G 1 L V WK H P L G S R Q W H U V H F W D W

THE 2012-2013 KENNESAW STATE UNIVERSITY
HIGH SCHOOL MATHEMATICS COMPETITION
PART II

SOLUTIONS

D HQVWRVKRZWKDWWKHUHLVSRLQW y KLKSDVHVWKURKDOSDDROD
LZWKHTWLRQRIWKHIRU $y = x^2 + 2ax + a$ HW mQ n W
W

pQB

x = - — K y = — W —

E W $y = x^2 + 2ax + a$ W
Q $y = x + ax + a + a - a = x + a + a - a$ W
W - a a - a W
$y = f(x) = -x^2 - x$

W

$f(x) = \frac{x-}{x+}$ $f(x) = f(f(x)) = \frac{\left(\frac{x-}{x+}\right)-}{\frac{x-}{x+}+} = \frac{x-}{x}$,Q

$f(x) = f(f(x)) = \frac{\left(\frac{x-}{x}\right)-}{\frac{x-}{x}+} = \frac{x-}{x-}$ D $f(x) = \frac{}{-x}$ $f(x) = \frac{x+}{-x}$

$f(x) = x$ Q $f(x) = \frac{x-}{x+}$ W $f_{6n}(x) = x$ Q

W f $= f$ $= \frac{-}{}$ = ——

$\overline{}$

$\angle CMB = \angle ANB = a$

and $\angle MPN = \angle APC = d$. Using quadrilateral BMPN

$d = 227.0$

Sind = sa2n (=2 3 0 na2 7 0 cos 2sa2=0iscosns2

Without loss of generality, let CB = 2, so

and $CM = \sqrt{5}$.

Using DMBC, $\sin a = \frac{2}{\sqrt{5}}$. Therefore, using the

appropriate double-angle formula, $\cos 2a = 1 - 2\sin^2 a = 1 - 2\left(\frac{2}{\sqrt{5}}\right)^2 = \frac{3}{5}$.

a = 1 Therefore $\sin d = \frac{3}{5}$

Since $\overline{CN}$ and $\overline{AM}$ are medians of triangle ABC, point of the triangle, so that AP = CP = $\frac{2}{3}$ CM =